

Gauged Lifshitz scalar field theories in two dimensions

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Abstract

We present two-dimensional gauged Lifshitz scalar field theories by considering the duality relation between the source current and the Noether current. Requiring the duality partially, we obtain a gauged model which recovers the bosonized Schwinger model for the IR limit. For the exact duality, however, the source current is not conserved, which means that the resulting theory is anomalous, so that the number of degrees of freedom is increased. The second model is consistently formulated by adding the Wess-Zumino type action to maintain the gauge invariance.

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I. INTRODUCTION

A Lifshitz scalar field theory [1–3] has been studied in the condensed matter physics as a description of tricritical phenomena involving spatially modulated phases. The Lifshitz index z reflects the anisotropic scaling between space and time, $x \rightarrow bx$ and $t \rightarrow b^z t$, and the Lifshitz scalar theory describes a free field fixed point $z = 2$ in four dimensions. Adding a relevant deformation can make the theory flow to a fixed point $z = 1$, which is in fact satisfied by the relativistic theory so that the Lorentz invariance emerging as an accidental symmetry at long distances. Recently, a renormalizable theory of gravity has been suggested by Hořava, as called Hořava-Lifshitz gravity [4, 5]. Due to the difference between the scaling dimensions of space and time, the Lagrangian is composed of usual second derivatives with respect to time and higher-order derivatives with respect to space. Of course, in the UV region, the explicit Lorentz covariance is broken.

On the other hand, a two-dimensional Dirac fermion coupled to the gauge field can be completely bosonized [6–9], and thus Schwinger model(SM) can be remarkably described by a scalar field coupled to a gauge field, which gives a single massive mode [6–11]. By the way, from the beginning, it is possible to realize this model in the bosonic regime. First, from the Noether current associated with the global symmetry, one can define the gauge invariant sector of the Noether current, though it is no longer conserved due to the axial anomaly when we consider the gauge field coupling. Then, the dual current through the duality relation of $J_V^\mu = \epsilon^{\mu\nu} J_\nu^A$ turns out to be nothing but the conserved gauge invariant vector current which can couple to the gauge field, where J_V^μ and J_ν^A are the gauge invariant conserved vector current and the gauge invariant non-conserved axial current, respectively, and $\epsilon_{01} = +1$. The vector current as a source current can couple to the gauge field, eventually, which yields the SM [10].

Now, one considers a single Lifshitz scalar field which recovers the Lorentz covariance for the IR limit, then it is natural to extend this free Lifshitz field theory to the gauged Lifshitz scalar field theory which gives the SM for the IR limit. In this paper, we would like to construct a gauged Lifshitz scalar field theories in two dimensions adopting the above-mentioned method. For this purpose, we define Noether and source currents for convenience, where they become the genuine axial and vector currents in QED for some limits. We first find the gauge invariant Noether current corresponding to the axial current

in section II. Then, the conserved source current can be obtained by imposing the duality relation partially. Then, we shall obtain the gauged Lifshitz scalar field theory which is continuously interpolated with the SM along with some conditions for the boundedness of the Hamiltonian. For the IR limit, we can recover the well-known SM. In section III, we study a Lifshitz scalar field coupled to the source current satisfying the full duality of $J_S^\mu = \epsilon^{\mu\nu} J_\nu^N$, where $J_V^\mu = \epsilon^{\mu\nu} J_\nu^A$ for the IR limit. But the source current is not conserved, so that the theory becomes anomalous. Moreover, there does not exist a bounded Hamiltonian, which breaks unitarity. So, we introduce an additional constraint to resolve the unitarity problem, and add the Wess-Zumino(WZ) action to recover the local gauge invariance [12–17]. The resulting theory is consistent, but the model is different from the case of the partial duality. Finally, summary is given in section IV.

II. GAUGED LIFSHITZ SCALAR WITH THE PARTIAL DUALITY

The action describing a Lifshitz scalar ϕ up to the fourth derivatives is defined by

$$S_L = \frac{1}{2} \int d^2x \left(\partial_\mu \phi \partial^\mu \phi + \beta \phi''^2 \right), \quad (1)$$

where $x^\mu = (t, x)$, and the Minkowski metric is $\eta_{00} = 1$. The parameter β is an arbitrary constant, which can be fixed based on physical requirements in later. The overdot and the prime denote the derivatives with respect to t and x , respectively. Introducing an auxiliary field λ for conveniences, the action (1) can be written as

$$S_L = \frac{1}{2} \int d^2x \left[\partial_\mu \phi \partial^\mu \phi + \beta (\lambda^2 - 2\lambda' \phi') \right]. \quad (2)$$

Now, the equations of motion are obtained as

$$\lambda = -\phi'', \quad (3)$$

$$\square \phi = \beta \lambda'', \quad (4)$$

where $\square = \partial_\mu \partial^\mu$. Since the action (2) is invariant under the global transformation given by $\delta \phi = \text{const.}$ and $\delta \lambda = 0$, the Noether current can be calculated as

$$J_N^\mu = \partial^\mu \phi - \beta \delta_1^\mu \lambda', \quad (5)$$

which is so-called the axial current in the bosonized SM for $\beta = 0$ [6–9]. Similarly to the SM, let us define the source current which couples to the gauge field, then it may be defined

by the duality relation between the source current and the Noether current as

$$J_S^\mu = \epsilon^{\mu\nu} J_\nu^N. \quad (6)$$

The reason why we want to define the source current in terms of the duality relation is that it should be coincident with the vector current in the bosonized SM for $\beta \rightarrow 0$, for instance, $J_V^\mu = \epsilon^{\mu\nu} J_\nu^A$. However, it breaks the conservation law as seen from $\partial_\mu J_S^\mu = -\beta \dot{\lambda}' \neq 0$.

In what follows, the source current satisfying the conservation law can be defined by requiring the dual relation partially between the source and the Noether currents; type I ($J_S^0 = \epsilon^{01} J_1^N$ but $J_S^1 \neq \epsilon^{10} J_0^N$) and type II ($J_S^0 \neq \epsilon^{01} J_1^N$ but $J_S^1 = \epsilon^{10} J_0^N$). Then, the source current can be written as

$$J_S^\mu = \epsilon^{\mu\nu} \partial_\nu (\phi + b\lambda), \quad (7)$$

where $\partial_\mu J_S^\mu = 0$. The constant, b should be chosen as β for the type I or 0 for the type II. Then, we can write down the action:

$$S_{(\text{partial})} = S_L + \int d^2x (-e J_S^\mu A_\mu) + S_{\text{Max}}, \quad (8)$$

where $S_{\text{Max}} = \int d^2x (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu})$, and A_μ and $F_{\mu\nu}$ are the gauge field and the field strength, respectively. As a comment, the Noether current in the presence of the gauge field is modified as

$$j^\mu = J_N^\mu + e \epsilon^{\mu\nu} A_\nu, \quad (9)$$

which satisfies $\partial_\mu j^\mu = 0$. However, this Noether current is not invariant under the local gauge transformation given by $\delta A_\mu = \frac{1}{e} \partial_\mu \Lambda$, so that the gauge invariant current should be chosen as J_N^μ in Eq. (5). This gauge invariant current has an axial anomaly, *i.e.*, $\partial_\mu J_N^\mu = -e \epsilon^{\mu\nu} \partial_\mu A_\nu$ [10].

The equations of motion from the action (8) are obtained as

$$\square \phi - \beta \lambda'' + e \epsilon^{\mu\nu} \partial_\mu A_\nu = 0, \quad (10)$$

$$\beta(\lambda + \phi'') - b e \epsilon^{\mu\nu} \partial_\mu A_\nu = 0, \quad (11)$$

$$\partial_\mu F^{\mu\nu} + e \epsilon^{\mu\nu} \partial_\mu (\phi + b\lambda) = 0. \quad (12)$$

By eliminating the scalar field and the auxiliary field from Eqs. (10), (11), and (12), we get

$$\left[\square + \beta \partial_x^4 + \frac{e^2 (1 - b \partial_x^2)^2}{1 - b e^2} \right] * F = 0, \quad (13)$$

where $*F = \frac{1}{2}\epsilon^{\mu\nu}F_{\mu\nu}$, so that there exists only one non-tachyonic massive mode with mass $m^2 = e^2/(1 - be^2)$ for $b < 1/e^2$. Note that the mass of the physical field for the type II ($b = 0$) is compatible with that of the SM. As for the type I ($b = \beta$), the higher-derivative correction gives rise to higher massive state than that of the SM.

In order to examine the boundedness of the Hamiltonian, we perform the constraint analysis in terms of the Hamiltonian formulation. The conjugate momenta with respect to A_0 , A_1 , ϕ , and λ are given by

$$\Pi^0 = 0, \quad \Pi^1 = \dot{A}_1 - A'_0, \quad (14)$$

$$\Pi_\phi = \dot{\phi} - eA_1, \quad \Pi_\lambda = -beA_1, \quad (15)$$

respectively, from which we read the primary constraints as follows [18]:

$$\Omega_1 \equiv \Pi_0 \approx 0, \quad \Omega_2 \equiv \Pi_\lambda + beA_1 \approx 0. \quad (16)$$

Then, the primary Hamiltonian is obtained as

$$H_p = \int dx (\mathcal{H}_c + \eta_1 \Omega_1 + \eta_2 \Omega_2), \quad (17)$$

where the canonical Hamiltonian density is given by

$$\mathcal{H}_c = \frac{1}{2}(\Pi_\phi + eA_1)^2 + \frac{1}{2}\phi'^2 - \frac{1}{2}\beta(\lambda^2 - 2\lambda'\phi') - e(\phi' + b\lambda')A_0 + \frac{1}{2}(\Pi^1)^2 + \Pi^1 A'_0, \quad (18)$$

and η_i 's are the multiplier fields. From the time evolution of the primary constraints (16), we obtain the secondary constraints:

$$\Omega_3 \equiv (\Pi^1)' + e(\phi' + b\lambda') \approx 0, \quad (19)$$

$$\Omega_4 \equiv \beta(\lambda + \phi'') + be\Pi^1 \approx 0. \quad (20)$$

Since the constraint algebra is first class which reflects the local gauge symmetry, we can choose the Coulomb gauge:

$$\Omega_5 \equiv A'_1 \approx 0. \quad (21)$$

Through the time evolution of this gauge fixing condition (21), we obtain one more constraint:

$$\Omega_6 \equiv (\Pi^1)' + A''_0 \approx 0. \quad (22)$$

Consequently, the net degrees of freedom is two in the phase space, and the reduced Hamiltonian at the constraint surfaces is obtained as

$$H_{\text{red}} = \frac{1}{2} \int dx \left(\Pi_\phi^2 + \frac{e^2}{1 - be^2} \phi^2 + \frac{1 + be^2}{1 - be^2} \phi'^2 + \frac{\beta}{1 - be^2} \phi''^2 \right), \quad (23)$$

which is positive for $0 < \beta < 1/e^2$ for the type I ($b = \beta$) and $\beta > 0$ for the type II ($b = 0$).

As a result, the gauged Lifshitz field theory can be obtained from the coupling to the conserved source current satisfying the partial duality, which gives the well-defined SM for $\beta \rightarrow 0$. Next, we are going to study the gauge coupling to the source current defined by the exact duality.

III. GAUGED LIFSHITZ SCALAR WITH THE EXACT DUALITY

We consider a Lifshitz scalar coupled to the gauge field by requiring the exact duality, *i.e.*, $J_S^\mu = \epsilon^{\mu\nu} J_\nu^N$ with the Noether current given by Eq. (5), which gives $J_S^\mu = \epsilon^{\mu\nu} (\partial_\nu \phi - \eta_{\nu 1} \beta \lambda')$. As mentioned earlier, the source current is not conserved, which implies that the action is not gauge invariant. So, we have to add the WZ action to cancel the gauge noninvariance [12–17]. Then, the total action is assumed to be

$$\tilde{S}_{(\text{exact})} = S_L + \int d^2x (-e J_S^\mu A_\mu) + S_{\text{Max}} + \tilde{S}_{\text{WZ}}, \quad (24)$$

where

$$S_{\text{WZ}} = \int d^2x \beta \lambda' \dot{\theta}, \quad (25)$$

and θ is a scalar field. Then, the total action is invariant under the local gauge transformation implemented by $\delta A_\mu = \frac{1}{e} \partial_\mu \Lambda$ and $\delta \theta = -\Lambda$.

The Noether current in the presence of the gauge field given by $j^\mu = J_N^\mu + e \epsilon^{\mu\nu} A_\nu$ is not gauge invariant, and the gauge invariant sector of the Noether current of Eq. (5) is still anomalous, $\partial_\mu J_N^\mu = -e \epsilon^{\mu\nu} \partial_\mu A_\nu$. On the other hand, the source current with the help of the equation of motion from the WZ action is automatically conserved, $\partial_\mu J_S^\mu = 0$.

In order to get the reduced Hamiltonian and study the boundedness of it, let us first obtain the conjugate momenta as follows:

$$\Pi^0 = 0, \quad \Pi^1 = \dot{A}_1 - A'_0, \quad (26)$$

$$\Pi_\phi = \dot{\phi} - e A_1, \quad \Pi_\lambda = 0, \quad \Pi_\theta = \beta \lambda', \quad (27)$$

which yields three primary constraints:

$$\Omega_1 \equiv \Pi^0 \approx 0, \quad \Omega_2 \equiv \Pi_\lambda \approx 0, \quad \Omega_3 \equiv \Pi_\theta - \beta\lambda' \approx 0. \quad (28)$$

Then, the primary Hamiltonian in terms of the Legendre transformation is written as

$$H_p = \int dx \left(\mathcal{H}_c + \sum_{i=1}^3 \eta_i \Omega_i \right), \quad (29)$$

where the canonical Hamiltonian is given by

$$\mathcal{H}_c = \frac{1}{2}[(\Pi_\phi + eA_1)^2 + \phi'^2 - \beta(\lambda^2 - 2\lambda'\phi')] - e(\phi' + \beta\lambda')A_0 + \frac{1}{2}(\Pi^1)^2 + \Pi^1 A'_0. \quad (30)$$

Now, the gauge fixing condition is taken as

$$\Omega_4 \equiv \theta \approx 0. \quad (31)$$

Then, from the time evolution of the primary constraints and the gauge fixing condition, we obtain the secondary constraints as

$$\Omega_5 \equiv (\Pi^1)' + e(\phi' + \beta\lambda') \approx 0, \quad (32)$$

$$\Omega_6 \equiv \lambda + \phi'' - eA'_0 \approx 0. \quad (33)$$

By applying all constraints to the primary Hamiltonian, the reduced Hamiltonian can be obtained as

$$H_{\text{red}} = \frac{1}{2} \int dx \left\{ (\Pi_\phi + eA_1)^2 + \phi'^2 + \beta(\phi''^2 - e^2 A_0'^2) + e^2 [\phi - \beta(\phi'' - eA'_0)]^2 \right\}, \quad (34)$$

which is not positive definite unless $\beta \neq 0$ even in spite of the gauge invariance of the action. It means that the gauged Lifshitz theory satisfying the exact duality relation between the source current and the Noether current is not unitary. So, we need more elaborations to obtain a positive definite Hamiltonian in this case.

From the reduced Hamiltonian (34), one can find a constraint to make the Hamiltonian to be positive. If we take $\phi'' + eA'_0 = 0$, then the reduced Hamiltonian can be positive definite, however, it is still problematic since the gauge invariance of the total action is lost. To implement the constraint gauge invariant fashion, the WZ action to recover the local gauge symmetry should be modified. To implement the new constraint consistently, we write the

new total action as

$$S_{(\text{exact})} = S_L + \int d^2x (-e J_S^\mu A_\mu) + S_{\text{Max}} + S_{\text{cons}} + S_{\text{WZ}}, \quad (35)$$

$$S_{\text{cons}} = \int d^2x \beta \xi' (\phi' + e A_0), \quad (36)$$

$$S_{\text{WZ}} = \tilde{S}_{\text{WZ}} + \int d^2x \beta \xi' \dot{\theta}, \quad (37)$$

where the new constraint was implemented by an auxiliary field, ξ , in Eq. (36). In the last term (37), the first one is the original WZ action (25) and the second one is due to the symmetry breaking of Eq. (36). Similarly to the previous case, the conserved Noether current $j^\mu = J_N^\mu + e\epsilon^{\mu\nu} A_\nu + \beta\delta_1^\mu \xi'$ is not gauge invariant. The gauge invariant sector of it is the same as Eq. (5), which is not conserved, $\partial_\mu J_N^\mu = -e\epsilon^{\mu\nu} \partial_\mu A_\nu - \beta\xi''$.

Now, the equations of motion for $\beta \neq 0$ are obtained as

$$\square\phi + e\epsilon^{\mu\nu} \partial_\mu A_\nu - \beta(\lambda'' - \xi'') = 0, \quad (38)$$

$$\lambda + \phi'' - eA'_0 - \dot{\theta}' = 0, \quad (39)$$

$$\phi'' + eA'_0 + \dot{\theta}' = 0, \quad (40)$$

$$\dot{\lambda}' + \dot{\xi}' = 0, \quad (41)$$

$$\partial_\mu F^{\mu\nu} = -e [\epsilon^{\mu\nu} \partial_\mu \phi + \beta\epsilon^{1\nu} (\lambda' + \xi')] . \quad (42)$$

For the explicit counting of degrees of freedom, we perform the constraint analysis. Then, we get four primary constraints as

$$\Omega_1 \equiv \Pi^0 \approx 0, \quad \Omega_2 \equiv \Pi_\lambda \approx 0, \quad (43)$$

$$\Omega_3 \equiv \Pi_\xi \approx 0, \quad \Omega_4 \equiv \Pi_\theta - \beta(\lambda' + \xi') \approx 0, \quad (44)$$

where Π_ξ and Π_θ are the momenta of ξ and θ , respectively. By the use of the Legendre transformation, the primary Hamiltonian can be obtained as

$$H_p = \int dx \left(\mathcal{H}_c + \sum_{i=1}^4 \eta_i \Omega_i \right), \quad (45)$$

where the canonical Hamiltonian is

$$\begin{aligned} \mathcal{H}_c = & \frac{1}{2} [(\Pi_\phi + eA_1)^2 + \phi'^2 - \beta(\lambda^2 - 2\lambda'\phi')] - e(\phi' + \beta\lambda')A_0 + \frac{1}{2}(\Pi^1)^2 + \Pi^1 A'_0 \\ & - \beta\xi'(\phi' + eA_0), \end{aligned} \quad (46)$$

and η_i 's are multiplier fields. Next, we choose the gauge fixing condition as

$$\Omega_5 \equiv \theta \approx 0. \quad (47)$$

Then, the time evolution of the four primary constraints and the gauge fixing condition yield the following secondary constraints,

$$\Omega_6 \equiv (\Pi^1)' + e[\phi' + \beta(\lambda' + \xi')] \approx 0, \quad (48)$$

$$\Omega_7 \equiv \lambda + \phi'' - eA'_0 \approx 0, \quad \Omega_8 \equiv \phi'' + eA'_0 \approx 0, \quad (49)$$

and the Lagrange multipliers are completely fixed.

The Dirac bracket [18] between two fields $\mathcal{A}(x)$ and $\mathcal{B}(y)$ is also defined by

$$\{\mathcal{A}(x), \mathcal{B}(y)\}_D \equiv \{\mathcal{A}(x), \mathcal{B}(y)\} - \sum_{i,j} \int dz dw \{\mathcal{A}(x), \Omega_i(z)\} \Delta_{ij}^{-1}(z, w) \{\Omega_j(w), \mathcal{B}(y)\}, \quad (50)$$

where $\Delta_{ij}^{-1}(x, y)$ is the inverse of $\Delta_{ij}(x, y)$ defined by $\Delta_{ij}(x, y) = \{\Omega_i(x), \Omega_j(y)\}$. The non-vanishing Dirac brackets can be calculated as

$$\{A_0(x), \Pi_\phi(y)\}_D = -\frac{1}{e}\delta'(x-y), \quad \{A_1(x), \xi(y)\}_D = -\frac{1}{\beta e}\delta(x-y), \quad (51)$$

$$\{A_1(x), \Pi^1(y)\}_D = \delta(x-y), \quad \{A_1(x), \Pi_\theta(y)\}_D = \frac{1}{e}\delta'(x-y), \quad (52)$$

$$\{\phi(x), \Pi_\phi(y)\}_D = \delta(x-y), \quad \{\lambda(x), \Pi_\phi(y)\}_D = -2\delta''(x-y), \quad (53)$$

$$\{\xi(x), \Pi_\phi(y)\}_D = -\frac{1}{\beta}\delta(x-y) + 2\delta''(x-y), \quad \{\Pi_\phi(x), \Pi_\theta(y)\}_D = -\delta'(x-y). \quad (54)$$

Note that they are well-defined for $\beta \neq 0$.

Now, the reduced Hamiltonian taking into account all constraint is obtained as

$$H_{\text{red}} = \frac{1}{2} \left[(\Pi_\phi + eA_1)^2 + \left(\Pi^1 - \frac{1}{e}\phi'' \right)^2 + 3\phi'^2 + \frac{4\beta e^2 - 1}{e^2} \phi''^2 \right], \quad (55)$$

which can be positive definite for $\beta \geq 1/(4e^2)$. Note that for $\beta \rightarrow 0$, the system does not go back to the reduced Hamiltonian of the SM because the constraint system for $\beta \neq 0$ is different from that for $\beta = 0$. Actually, there exist more degrees of freedom compared to the case of the SM. This fact can be easily seen by solving Eqs. (38)–(42),

$$(\square + 4\beta\partial_x^4 + e^2) * F = \mathcal{J}, \quad \square \mathcal{J} = 0, \quad (56)$$

where the field \mathcal{J} is related to the original fields by $\mathcal{J} = (\square - \partial_x^2 + 4\beta\partial_x^4) (*F - e\phi)$. Note that there are two modes: one is a massless mode and the other is a massive mode with

mass e^2 with the modified dispersion relation. As a result, the degrees of freedom is not conserved for $\beta \rightarrow 0$, which implies that the SM can not be reproduced for the IR limit in spite of the consistent formulation.

IV. CONCLUSION

In summary, we have presented two gauged Lifshitz scalar field theory models. Actually, our criteria is just whether the duality between the source current and the Noether current is preserved or not, which was clearly satisfied in the SM. For the case of the partial duality in section II, the SM limit is well-defined, so that the massive single mode survives with the positivity of the Hamiltonian. In this case, the characteristic parameter “ β ” of the higher-derivative in the Lifshitz scalar field theory can increase the effective mass, where the lower bound is e^2 . This model is plausible in the sense that it is continuously interpolated with the SM, but, unfortunately, the duality relation is not complete. The second model in section III seems to be nice in that the duality relation is preserved, but it can not be smoothly connected with the SM. To get the SM in the second case, we have to begin with $\beta = 0$. The final result shows that there exist one massive and one massless modes. It is rather close to the chiral SM [19–23] from the view-point of degrees of freedom. Actually, the additional massless degree of freedom is due to the non-conservation of the source current, which spoils the local gauge invariance and the more degree of freedom survives.

So far, we have discussed the gauged Lifshitz scalar field theories based on the duality relation. Strictly speaking, they are not unique, so it will be interesting to find the other Lifshitz scalar models to match the SM for the IR limit without resort to the duality relation, and discuss some physical applications. We hope this issue will be addressed elsewhere.

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